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August 1988

**ORBIT DETERMINATION AND ANALYSIS
OF COSMOS 220 ROCKET AT RESONANCE,
TO EVALUATE 29TH-ORDER HARMONICS**

by

Doreen M. C. Walker

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R O Y A L A E R O S P A C E E S T A B L I S H M E N T

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ORBIT DETERMINATION AND ANALYSIS OF COSMOS 220 ROCKET
AT RESONANCE, TO EVALUATE 29TH-ORDER HARMONICS

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Doreen M. C. Walker

SUMMARY

The orbital parameters of Cosmos 220 rocket (1968-40B) have been determined at 74 epochs from some 6300 observations. For 38 of the determinations, Hewitt camera observations were available.

The orbit was determined from January 1983 until July 1985, and during this time the satellite passed through 29:2 resonance. The variations in inclination and eccentricity have been analysed to determine six lumped geopotential harmonics of order 29. The first two have standard deviations equivalent to accuracies of 0.3 and 0.6 cm in geoid height, better than has previously been determined for order 29 by resonance analysis or any other method. The standard deviations of the other four values correspond to accuracies between 1.2 and 3.5 cm in geoid height. The results are independent of recent comprehensive geoid models and are used to test five such models: the comparisons show that the estimated errors given for the models are quite realistic.

Departmental Reference: Special Systems 8

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1 INTRODUCTION

The rocket which put Cosmos 220 into orbit on 1968 May 7 itself also entered orbit and was designated 1968-40B. Initially it had the following orbital elements¹: inclination 74.05° ; nodal period 99.17 min; apogee and perigee heights, 759 km and 678 km respectively; and eccentricity 0.006.

After some 15 years in orbit, 1968-40B was slowly approaching the condition of 29:2 resonance, *ie* when the satellite makes 29 revolutions while the Earth spins twice, relative to the orbital plane. As a result 1968-40B was, in 1983, placed on the priority list for high-priority observing and remained there for three years, so that the orbit of the satellite could be determined at frequent intervals while it was passing through the resonance. In this Report the orbit of 1968-40B has been determined from radar and optical observations between 1983 January and 1985 July using the RAE orbit determination program PROP in the PROP6 version². The inclination and eccentricity were then analysed over this time to evaluate 29th-order lumped harmonics.

2 THE OBSERVATIONS, ORBITS AND OBSERVATIONAL ACCURACY

2.1 The observations

The orbit of 1968-40B has been determined at 74 epochs, from some 6300 observations, between 1983 January 5 and 1985 July 27. The number of observations used in each determination is given in Table 1 and the source of the observations is also indicated.

There were four groups of observations available. Those listed first in Table 1 were the most accurate, being made by the University of Aston's Hewitt cameras at the Royal Greenwich Observatory site at Herstmonceux and at the Siding Spring Observatory site in Australia: these observations usually have an accuracy of a few seconds of arc. The second consisted of visual observations made by volunteer observers who reported to the University of Aston, and these observations usually have accuracies between 2 and 5 minutes of arc in good observing conditions. The third group is made up of Navspasur observations kindly supplied by the US Naval Research Laboratory, with accuracies of about 2 minutes of arc. The final group, British radar observations, came from the radar trackers at RAF, Fylingdales.

2.2 The orbits

The orbits were determined at approximately 14-day intervals with the aid of the RAE orbit refinement program PROP in the PROP 6 version, and the

Table 1

Sources of the observations used on each orbit

Orbit No.	Source of observation				Total	Orbit No.	Source of observation				Total
	Hewitt camera	Visual	US Navy	British radar			Hewitt camera	Visual	US Navy	British radar	
1		3	70	27	100	38	25H	10	25	21	81
2		11	61	28	100	39	10H	7	18	12	47
3		1	66	33	100	40		13	41	36	90
4		12	61	26	99	41		13	50	31	94
5	12S	2	48	14	76	42			40	31	71
6		5	60	17	82	43	10H		37		47
7	12H	3	12	56	83	44	10H	7	27	6	50
8	6H		53	15	74	45	10H	21	21	17	69
9	10H	8	68	13	99	46	5H	9	23	20	57
10	6H		64	6	76	47		1	36	33	70
11	17H	7	47	29	100	48		2	43	38	83
12		6	47	31	84	49		8	40	34	82
13		1	55	44	100	50		5	54	38	97
14	20H		36	37	93	51	6S	8	55	31	100
15	21H	3	31	26	81	52		24	49	27	100
16	11H	3	26	23	63	53	6S		52	42	100
17	6H	6	35	19	66	54	4S		65	26	95
18	5H	14	25	27	71	55			48	42	90
19		32	42	26	100	56	14S	23	48	15	100
20		10	50	39	99	57	6S	4	48	40	98
21			56	36	92	58		4	45	51	100
22	3H	8	54	30	95	59	6S	3	59	31	99
23	6H	2	36	20	64	60		21	52	27	100
24	6H	6	24	20	56	61	7S		49	43	99
25	6H		31	18	55	62	9S		27	31	67
26		2	40	36	78	63			53	47	100
27		2	39	40	81	64		17	55	28	100
28		5	29	36	70	65	6S	4	62	28	100
29		7	53	40	100	66	4S	1	39	36	80
30	9H	5	36	24	74	67		2	52	45	99
31		6	46	46	98	68	5H	1	65	29	100
32			48	42	90	69		14	53	33	100
33		11	50	39	100	70	12S	9	36	39	96
34		4	47	37	88	71			50	50	100
35			44	29	73	72		4	54	42	100
36	20H		35		55	73	8H	11	35	42	96
37	20H		37	10	67	74	4S	1	43	36	84

S = camera at Siding Spring, Australia. H = camera at Herstmonceux, England

orbital elements at each epoch are listed in Table 2, with the standard deviations below each value. The epoch for each orbit is at 00 hours on the day indicated, and the PROP program fits the mean anomaly M by a polynomial of the form

$$M = M_0 + M_1 t + M_2 t^2 + M_3 t^3 + M_4 t^4 + M_5 t^5, \quad (1)$$

where t is the time measured from epoch, and the number of M coefficients used depends on the drag. The satellite 1968-40B was in a nearly circular orbit at a height of about 650 km, where drag is slight: consequently on 31 orbits it was only necessary to use the coefficients M_0 and M_1 ; on 39 orbits the coefficients $M_0 - M_2$ were needed; and on 4 orbits the coefficients $M_0 - M_3$ were required.

The value of ϵ , the parameter indicating the measure of fit of the observations to the orbit, varied between 0.44 and 1.05, and had an average value of 0.74. For all 74 orbits the standard deviations in inclination were between 0.0005° and 0.0014° , the average being 0.0009° which is equivalent to about 110 m in distance. The values of the standard deviations in eccentricity varied between 0.000001 and 0.000022, and the average value was 0.000011, corresponding to 80 m in distance. The accuracy for the right ascension of the node was nearly the same as that for the inclination.

2.3 The accuracy of the observations

The 3311 US Navy observations used in the orbit determination were of the usual consistent accuracy of about 2 minutes of arc, and those of the British radars were also of their customary standard. The residuals of other observations have been obtained with the aid of the ORES computer program³ and sent to observers. Table 3 lists the rms residuals of the observing stations with five or more observations accepted in the final orbit determinations.

The residuals of some visual observers are a little higher than usual, but as these rms values include all observations made by each observer, with no attempt to eliminate those made in bad conditions of visibility, this is not surprising.

The Hewitt cameras did very well: the Herstmonceux camera contributed 265 observations with rms residual of 8 seconds of arc, equivalent to about 30 m in position, and the Siding Spring camera contributed 81 observations with rms residual of 3 seconds of arc, equivalent to about 10 m. Many of the 74 orbits

Table 2

Values of orbital parameters at 74 epochs with standard deviations

	MJD	Date	a	e	i	Ω	ω	$\omega + M_0$	M_1	M_2	M_3	e	N	D
1	45339	1983 Jan 5	7067.0870 2	0.005515 11	74.0474 12	28.877 1	39.9 1	243.858	5262.0693 3	0.0036 1		0.55	77	7.9
2	45351	1983 Jan 17	7066.9854 3	0.005118 16	74.0476 11	5.931 1	15.9 1	3.338	5262.1828 3	0.0040 2		0.70	86	7.9
3	45366	1983 Feb 1	7066.8993 2	0.004618 14	74.0522 12	337.247 1	343.2 1	64.240	5262.2792 3	0.0038 1		0.57	84	7.6
4	45386	1983 Feb 21	7066.7927 2	0.004064 8	74.0497 9	299.006 1	293.8 2	147.995	5262.3982 2	0.0015 1		0.51	89	8.4
5	45401	1983 Mar 8	7066.7328 4	0.004095 15	74.0459 12	270.318 1	258.9 2	211.848	5262.4649 4	0.0013 1		1.05	75	7.1
6	45418	1983 Mar 25	7066.6701 2	0.004381 10	74.0448 8	237.805 1	211.0 1	357.415	5262.5349 2	0.0033 1		0.44	65	9.5
7	45430	1983 Apr 6	7066.6081 1	0.004786 11	74.0477 6	214.857 1	183.6 1	122.248	5262.6044 1	0.0015 1		0.47	73	7.5
8	45438	1983 Apr 14	7066.5794 3	0.005067 9	74.0497 6	199.558 1	166.5 1	85.849	5262.6366 3	0.0026 1		0.46	58	7.5
9	45455	1983 May 1	7066.4942 3	0.005597 7	74.0491 5	167.049 1	133.2 1	234.604	5262.7318 3	0.0025 1	0.00016 3	0.56	90	9.7
10	45480	1983 May 26	7066.3749 2	0.005879 6	74.0450 7	119.231 1	87.0 1	350.230	5262.8649 2	0.0035 1		0.54	74	9.9
11	45496	1983 Jun 11	7066.3370 5	0.005717 8	74.0496 6	88.629 1	57.6 1	282.015	5262.9410 6	0.0022 1	0.00029 6	0.57	99	7.5
12	45508	1983 Jun 23	7066.2519 10	0.005449 18	74.0486 12	65.675 1	34.9 1	51.656	5263.0025 11	0.0047 2	0.00057 11	0.77	77	7.6
13	45527	1983 Jul 12	7066.2061 2	0.004838 12	74.0459 8	29.332 1	355.4 1	288.039	5263.0535 2	0.0009 1		0.69	58	7.9
14	45550	1983 Aug 4	7066.1584 1	0.004214 9	74.0482 8	345.327 1	300.1 1	328.944	5263.1069 1	0.0008 2	0.00028 8	0.74	91	5.7
15	45560	1983 Aug 14	7066.1272 1	0.004092 10	74.0531 9	326.203 1	273.3 2	18.466	5263.1420 1			0.89	81	3.8
16	45564	1983 Aug 18	7066.1229 1	0.004075 8	74.0502 11	318.547 2	262.0 2	182.379	5263.1468 1			0.87	61	3.7
17	45573	1983 Aug 27	7066.0888 7	0.004182 10	74.0506 8	301.333 1	239.5 2	11.371	5263.1849 8			0.66	63	4.0
18	45577	1983 Aug 31	7066.0803 8	0.004267 12	74.0492 10	293.678 2	228.9 2	175.454	5263.1943 9			0.82	68	3.8
19	45584	1983 Sep 7	7066.0630 5	0.004532 16	74.0466 10	280.289 1	211.5 2	282.683	5263.2135 6	0.0016 4		0.80	99	5.0
20	45595	1983 Sep 18	7066.0336 4	0.004938 19	74.0436 10	259.240 1	186.4 1	194.332	5263.2462 4	0.0019 3		0.82	99	6.0
21	45610	1983 Oct 3	7065.9964 3	0.005404 14	74.0460 9	230.543 1	154.9 1	270.777	5263.2880 3	0.0017 2		0.67	89	6.8
22	45635	1983 Oct 28	7065.9138 4	0.005956 7	74.0468 7	182.716 1	107.3 2	39.977	5263.3803 5	0.0010 3		0.81	95	5.7
23	45642	1983 Nov 4	7065.8915 8	0.005982 10	74.0479 7	169.326 1	94.3 2	148.519	5263.4053 9	0.0026 7		0.83	64	4.0
24	45646	1983 Nov 8	7065.8834 7	0.005985 9	74.0462 6	161.672 1	86.8 1	313.466	5263.4142 7	0.0045 7		0.69	55	3.7
25	45650	1983 Nov 12	7065.8670 7	0.005974 8	74.0464 7	154.020 1	78.9 2	118.470	5263.4326 8	0.0050 4		0.65	55	4.0
26	45662	1983 Nov 24	7065.8432 4	0.005859 14	74.0426 11	131.058 1	57.2 2	253.805	5263.4590 5			0.83	78	5.8
27	45677	1983 Dec 9	7065.8172 5	0.005430 21	74.0461 13	102.358 2	28.6 2	333.335	5263.4883 6	0.0012 4		0.93	79	5.8
28	45697	1983 Dec 29	7065.7800 6	0.004675 22	74.0451 14	64.089 2	345.4 2	80.175	5263.5298 7			0.96	70	5.6
29	45705	1984 Jan 6	7065.7664 4	0.004469 13	74.0462 10	48.783 1	326.8 1	51.101	5263.5450 4	0.0008 2		0.83	97	6.3
30	45719	1984 Jan 20	7065.7425 3	0.004236 10	74.0441 9	21.992 1	291.2 3	270.537	5263.5717 3			0.77	70	6.0
31	45730	1984 Jan 31	7065.7154 4	0.004147 9	74.0444 9	0.942 1	261.8 2	186.090	5263.6019 5	0.0010 2		0.71	97	6.6
32	45748	1984 Feb 18	7065.6665 3	0.004498 16	74.0470 11	326.500 1	215.7 2	212.329	5263.6567 4	0.0013 2		0.82	90	7.7
33	45764	1984 Mar 5	7065.6115 3	0.005026 20	74.0448 10	295.884 1	179.5 1	156.558	5263.7181 4	0.0022 2		0.85	98	6.8
34	45774	1984 Mar 15	7065.5813 4	0.005341 9	74.0409 10	276.748 1	158.6 1	212.213	5263.7516 5	0.0022 3		0.76	84	5.7
35	45795	1984 Apr 5	7065.4531 3	0.005867 11	74.0426 10	236.553 1	117.9 1	186.768	5263.8951 4	0.0041 2		0.65	70	7.5
36	45803	1984 Apr 13	7065.4144 1	0.005946 8	74.0472 9	221.245 1	103.3 1	160.781	5263.9385 1			0.68	54	5.3
37	45812	1984 Apr 22	7065.3851 1	0.005941 6	74.0496 7	204.024 1	86.5 1	356.832	5263.9714 1	0.0019 1		0.69	66	5.5

Table 2 (concluded)

	MJD	Date	a	e	i	Ω	ω	$\omega + M_0$	M_1	M_2	M_3	ϵ	N	D
38	45817	1984 Apr 27	7065.3669 1	0.005920 6	74.0480 8	194.453 1	77.5 2	25.887	5263.9917 1	0.0025 1		0.92	79	4.0
39	45821	1984 May 1	7065.3534 1	0.005888 5	74.0463 13	186.798 2	70.0 2	193.207	5264.0068 1			0.84	47	3.5
40	45826	1984 May 6	7065.3397 5	0.005820 13	74.0426 12	177.232 2	61.5 2	222.425	5264.0218 5	0.0014 3		0.90	89	5.8
41	45834	1984 May 14	7065.3224 3	0.005625 11	74.0436 8	161.924 1	46.2 1	197.304	5264.0412 3	0.0014 2		0.62	92	5.8
42	45853	1984 Jun 2	7065.2512 5	0.004985 18	74.0409 10	125.558 1	7.9 1	93.605	5264.1207 5	0.0016 3		0.69	70	5.7
43	45861	1984 Jun 10	7065.2279 5	0.004761 8	74.0463 10	110.245 1	350.0 1	69.309	5264.1470 5	0.0022 2		0.64	47	5.6
44	45866	1984 Jun 15	7065.2162 2	0.004572 8	74.0469 11	100.680 2	338.2 1	99.217	5264.1600 2			0.72	49	3.7
45	45870	1984 Jun 19	7065.2083 1	0.004411 17	74.0465 9	93.023 1	328.7 2	267.192	5264.1689 1			0.80	66	3.7
46	45878	1984 Jun 27	7065.1916 6	0.004262 11	74.0477 13	77.712 1	309.4 2	243.255	5264.1877 7			0.73	53	3.0
47	45897	1984 Jul 16	7065.1647 5	0.004077 12	74.0422 13	41.343 2	259.2 3	141.867	5264.2175 6			0.87	70	5.2
48	45906	1984 Jul 25	7065.1542 4	0.004192 13	74.0415 11	24.116 1	236.0 2	340.336	5264.2291 5			0.87	83	5.9
49	45934	1984 Aug 22	7065.1208 4	0.005170 19	74.0489 10	330.524 1	170.6 1	78.463	5264.2669 4			0.73	79	5.3
50	45947	1984 Sep 4	7065.1034 3	0.005569 14	74.0487 9	305.644 1	144.5 1	85.825	5264.2863 4			0.73	94	5.8
51	45955	1984 Sep 12	7065.0897 3	0.005168 6	74.0445 6	290.332 1	129.3 1	62.806	5264.3015 4	0.0007 2		0.59	58	5.8
52	45964	1984 Sep 21	7065.0783 3	0.005916 8	74.0426 7	273.106 1	112.8 1	262.030	5264.3141 3	0.0014 2		0.58	96	6.2
53	45980	1984 Oct 7	7065.0527 3	0.005940 6	74.0446 6	242.473 1	82.8 1	216.544	5264.3428 3	0.0010 2		0.60	98	6.0
54	45986	1984 Oct 13	7065.0438 2	0.005878 4	74.0482 6	230.989 1	71.8 1	109.605	5264.3529 3	0.0008 2		0.60	89	5.6
55	46002	1984 Oct 29	7065.0184 4	0.005571 14	74.0486 9	200.369 1	41.9 1	64.741	5264.3814 5	0.0008 3		0.73	87	5.6
56	46017	1984 Nov 13	7065.0017 1	0.005091 1	74.0460 6	171.662 1	11.5 1	158.028	5264.3999 1			0.59	97	5.2
57	46038	1984 Dec 4	7064.9680 4	0.004338 17	74.0415 9	131.464 1	322.7 1	145.165	5264.4374 4			0.85	98	5.8
58	46056	1984 Dec 22	7064.9546 3	0.004092 10	74.0504 10	97.011 1	276.6 2	186.080	5264.4527 3	0.0003 1		0.80	99	8.3
59	46077	1985 Jan 12	7064.9318 4	0.004434 6	74.0435 10	56.811 1	222.4 1	174.259	5264.4779 5			0.86	99	6.5
60	46091	1985 Jan 26	7064.9176 2	0.004829 14	74.0463 7	30.014 1	189.4 1	46.650	5264.4939 3	0.0004 2		0.57	96	6.3
61	46104	1985 Feb 8	7064.9057 2	0.005270 5	74.0445 5	5.123 1	162.0 1	56.926	5264.5072 2	0.0008 3		0.89	93	5.9
62	46109	1985 Feb 13	7064.9007 2	0.005423 6	74.0473 10	355.554 1	151.8 1	88.619	5264.5129 2			0.86	67	4.0
63	46121	1985 Feb 25	7064.8919 5	0.005761 15	74.0489 11	332.591 1	128.4 2	236.776	5264.5229 6			0.90	97	4.6
64	46134	1985 Mar 10	7064.8821 4	0.005964 10	74.0468 10	307.710 1	104.6 2	247.432	5264.5337 4			0.89	97	5.2
65	46144	1985 Mar 20	7064.8784 3	0.005998 5	74.0431 8	288.567 1	85.6 1	311.077	5264.4376 3			0.63	90	5.9
66	46154	1985 Mar 30	7064.8742 5	0.005865 6	74.0417 11	269.417 1	67.9 1	14.763	5264.5422 6			0.93	78	6.0
67	46170	1985 Apr 15	7064.8711 2	0.005541 15	74.0418 10	238.788 1	37.5 1	332.714	5264.5457 3			0.81	96	7.5
68	46182	1985 Apr 27	7064.8613 3	0.005208 15	74.0484 8	215.822 1	12.8 1	121.249	5264.5570 4			0.81	95	6.0
69	46195	1985 May 10	7064.8554 2	0.004703 13	74.0464 7	190.939 1	344.9 1	132.293	5264.5634 3			0.56	95	7.0
70	46205	1985 May 20	7064.8464 2	0.004390 6	74.0462 8	171.795 1	320.1 1	196.257	5264.5735 2	0.0012 1		0.85	96	6.9
71	46221	1985 Jun 5	7064.8382 3	0.004106 9	74.0433 10	141.164 1	279.2 3	154.757	5264.5825 4			0.84	100	7.5
72	46234	1985 Jun 18	7064.8289 3	0.004175 14	74.0450 11	116.276 1	244.3 2	166.178	5264.5930 4			0.88	98	7.4
73	46251	1985 Jul 5	7064.8232 3	0.004576 18	74.0489 8	83.734 1	202.7 1	347.410	5264.5996 4			0.88	96	6.4
74	46273	1985 Jul 27	7064.8101 5	0.005325 5	74.0451 12	41.614 1	154.8 1	201.001	5264.6141 6			0.93	84	5.3

KEY MJD modified Julian day
a semi major axis (km)
e eccentricity
i inclination (deg)
 Ω right ascension of ascending node (deg)
 ω argument of perigee (deg)
 M_0 mean anomaly at epoch (deg)
 M_1 mean motion n (deg/day)
 M_2, M_3 later coefficients in the polynomial for M
c measure of fit
N number of observations accepted in each orbit determination
D time covered by the observations (days)

Table 3

Residuals for observing stations with five or more observations
accepted in the final orbit determination

Station	Number of observations		Rms residuals Minutes of arc		
	Accepted	Rejected	RA	Dec	Total
414 Capetown	20	1	1.7	2.1	2.7
1184 Eilenburg	7	0	5.9	1.9	6.2
2265 Farnham	24	3	5.4	5.0	7.4
2392 Cowbeech	8	1	1.0	2.4	2.6
2414 Bournemouth	132	0	3.2	3.5	4.7
2418 Sunningdale	27	0	2.3	2.2	3.2
2420 Willowbrae	83	6	4.0	3.4	5.2
2430 Stevenage 4	8	0	1.7	2.1	2.7
2437 Warrington	5	0	6.1	7.4	9.6
2539 Dymchurch	25	0	1.5	1.6	2.2
2657 Bridgwater	30	0	1.8	2.2	2.8
2658 Hillsborough	8	0	3.0	1.6	3.4
2659 Herstmonceux 3 (Hewitt camera)	265	2	0.10	0.10	0.14
2665 Cluj 3	5	0	4.3	5.9	7.3
4156 Apeldoorn	7	0	1.2	3.8	4.0
9652 Siding Spring (Hewitt camera)	81	4	0.03	0.04	0.05

utilize more than one Hewitt camera transit and the PROP orbital model is not accurate to better than 50 m over an interval of more than a few days because lunisolar perturbations are neglected. So orbital error almost certainly contributes to the Hewitt camera residuals: the cameras are more accurate than the residuals indicate. The Herstmonceux camera suffered more from this effect, because many of the orbit determinations included multiple transits: for example, orbit 38 used four Hewitt camera plates at daily intervals.

3 EQUATIONS FOR 29:2 RESONANCE

The theory for 29:2 resonance is detailed in Ref 4, where all the parameters used are defined. The theoretical equation for the change in inclination at resonance, for orbits with small eccentricity, is

$$\begin{aligned}
\frac{di}{dt} = \frac{n}{\sin i} \left(\frac{R}{a} \right)^{29} & \left[\frac{R}{a} (29 - 2 \cos i) \bar{F}_{30,29,14} \left\{ \bar{S}_{29}^{0,2} \sin \phi + \bar{C}_{29}^{0,2} \cos \phi \right\} \right. \\
& + 16e(29 - \cos i) \bar{F}_{29,29,14} \left\{ \bar{C}_{29}^{1,1} \sin(\phi - \omega) - \bar{S}_{29}^{1,1} \cos(\phi - \omega) \right\} \\
& + 12e(29 - 3 \cos i) \bar{F}_{29,29,13} \left\{ \bar{C}_{29}^{-1,3} \sin(\phi + \omega) - \bar{S}_{29}^{-1,3} \cos(\phi + \omega) \right\} \\
& \left. + \text{terms in } e^{|q|} \frac{\cos(\gamma\phi - q\omega)}{\sin(\gamma\phi - q\omega)} \right], \quad \text{.....(2)}
\end{aligned}$$

$$\text{where} \quad \phi = 2(\omega + M) + 29(\Omega - \nu) \quad (3)$$

is the resonance angle, ω being the argument of perigee, M the mean anomaly, Ω the right ascension of the node and ν the sidereal angle. The quantities γ and q are integers, but only three terms are given explicitly in equation (2) because it is believed the others are small: terms with $q = \pm 2$ have e^2 as a multiplying factor and, for 1968-40B, e is 0.006 at most during the time of 29:2 resonance; while the terms with $\gamma = 2$ are associated with harmonics of order 58, which should be much smaller than those of order 29. The \bar{F} in equation (2) are functions of inclination only, defined in Ref 4.

The theoretical equation for the variation of e , for small e , is⁴

$$\begin{aligned}
\frac{de}{dt} = n \left(\frac{R}{a} \right)^{29} & \left[- \frac{R}{a} \bar{F}_{30,29,14} e \left(\bar{S}_{29}^{0,2} \sin \phi + \bar{C}_{29}^{0,2} \cos \phi \right) \right. \\
& - 16 \bar{F}_{29,29,14} \left\{ \bar{C}_{29}^{1,1} \sin(\phi - \omega) - \bar{S}_{29}^{1,1} \cos(\phi - \omega) \right\} \\
& + 12 \bar{F}_{29,29,13} \left\{ \bar{C}_{29}^{-1,3} \sin(\phi + \omega) - \bar{S}_{29}^{-1,3} \cos(\phi + \omega) \right\} \\
& \left. + \text{terms in } \left[e^{|q|-1} \left\{ q - (\gamma + q)e^2 \right\} \frac{\cos(\gamma\phi - q\omega)}{\sin(\gamma\phi - q\omega)} \right] \right] . \quad (4)
\end{aligned}$$

The main terms in equation (4) are expected to be those with $(\gamma, q) = (1, 1)$ and $(1, -1)$, but the $(\gamma, q) = (1, 0)$ term is included so that the analysis conforms with that for inclination.

The quantities such as $\bar{S}_{29}^{0,2}$ in equations (2) and (4) are lumped geopotential harmonics of order 29, which can be expressed as linear sums of individual coefficients $\bar{C}_{\ell m}$ and $\bar{S}_{\ell m}$,

$$\bar{c}_m^{q,l} = \sum_{\ell} Q_{\ell}^{q,k} \bar{c}_{\ell m} \quad \text{and} \quad \bar{s}_m^{q,k} = \sum_{\ell} Q_{\ell}^{q,k} \bar{s}_{\ell m}, \quad (5)$$

and these individual coefficients can be evaluated when enough lumped harmonics from satellites at different inclinations are available. The summation for ℓ in equation (5) is in steps of 2, beginning at the lowest ℓ , which is either 29 or 30 here (see Ref 4). Also $k = 2\gamma - q$ for 29:2 resonance.

The orders of magnitude of the terms in the equations for di/dt and de/dt can be estimated, as the $\bar{c}_{\ell m}$ (or $\bar{s}_{\ell m}$) are expected to be of order $10^{-5}/\ell^2$, so the value of $\bar{c}_m^{q,k}$ (or $\bar{s}_m^{q,k}$) can be taken to be of order $\left\{ \sum (Q_{\ell} \times 10^{-5}/\ell^2)^2 \right\}^{1/2}$, the Q coefficients being obtained for specific values of (γ, q) using the RAE computer program PROF.

Estimating the orders of magnitude of the terms in equation (2) for inclination suggests that, if just the $(\gamma, q) = (1, 0)$ term is used, the likely errors are 2% from the neglect of the $(2, 0)$ term and 8% from the neglect of the $(1, \pm 1)$ terms. In equation (4) the $(\gamma, q) = (1, 0)$ term only contributes about 0.5%. As mentioned before, the $q = 2$ terms contain a factor e^2 and therefore they will not be needed.

4 ANALYSIS OF INCLINATION AT 29:2 RESONANCE

Cosmos 220 rocket passed through exact 29:2 resonance on 1984 Aug 15 and its orbit has been determined either side of resonance, with the rate of change of resonance angle, $\dot{\phi}$, increasing from -4.3 to +0.7 deg/day. The variation of both $\dot{\phi}$ and the resonance angle ϕ , given by equation (3), are shown in Fig 1. The decrease of $\dot{\phi}$ became very slow in the last 18 months due to the reduction in solar activity.

Before the changes in inclination due to resonance can be analysed, all other known perturbations must be removed. The 74 values of inclination in Table 2 were therefore cleared of lunisolar and zonal harmonic perturbations, by using the computer program PROD⁵ with 1-day integration steps, and the perturbation due to the $J_{2,2}$ tesseral harmonic, which is recorded on each PROP run, was also removed from each value of inclination. Perturbations due to earth and ocean tides should not exceed 50 m and need not be considered, since the values of inclination have an average accuracy of 110 m, the most accurate being 0.0005° which is approximately equivalent to 65 m.

The values of inclination, cleared of the perturbations mentioned, and with the standard deviations quoted in Table 2, were fitted with equation (2) in integrated form using the RAE THROE⁶ computer program. This program removes the perturbations due to atmospheric rotation and lunisolar precession of the Earth's axis before fitting the remaining perturbation due to resonance. The density scale height, H , was taken as 85 km, appropriate to a height of 683 km, $0.4H$ above perigee⁷, and the atmospheric rotation rate, Λ , was taken⁸ as 0.9 rev/day. The values of M_2 were altered to mean values, \bar{M}_2 , by the technique described in Ref 9.

The first fitting of the values of inclination by THROE, with $(\gamma, q) = (1, 0)$, gave $\epsilon = 2.29$, where ϵ is the measure-of-fit parameter. On inspection of the PROP runs for some of the ill-fitting values of inclination, it was found that the runs included quite a few low-elevation radar observations, and relaxing the standard deviations of the range values had a beneficial effect on the values of inclination. After these relaxations the value of ϵ on the next THROE run was 1.081 and the values of the lumped harmonics were as follows:

$$10^9 \bar{C}_{29}^{0,2} = -2.6 \pm 0.5, \quad 10^9 \bar{S}_{29}^{0,2} = 8.9 \pm 1.0. \quad (6)$$

For this fitting, one value of inclination had its standard deviation increased by a factor of 1.5 to ensure that all weighted residuals were less than 2ϵ .

The addition of the $(\gamma, q) = (1, \pm 1)$ terms did not improve the standard deviation of $(\bar{C}, \bar{S})_{29}^{0,2}$ and the additional harmonics were not well determined. The values obtained from the THROE run were:

$$\left. \begin{array}{ll} \bar{C}_{29}^{0,2} = -2.2 \pm 0.5 & \bar{S}_{29}^{0,2} = 9.2 \pm 1.0 \\ \bar{C}_{29}^{1,1} = 15 \pm 13 & \bar{S}_{29}^{1,1} = 62 \pm 24 \\ \bar{C}_{29}^{-1,3} = -94 \pm 36 & \bar{S}_{29}^{-1,3} = -24 \pm 37 \end{array} \right\} \quad (7)$$

with $\epsilon = 0.927$. The values of $(\bar{C}, \bar{S})_{29}^{1,1}$ and $(\bar{C}, \bar{S})_{29}^{-1,3}$ were of the right order according to the $10^{-5}/\ell^2$ rule, but they were poorly determined. The results are given because the run was needed for the simultaneous fitting of inclination and eccentricity. The addition of the $(\gamma, q) = (2, 0)$ terms also proved unhelpful.

The values of inclination, cleared of all known perturbations except those due to resonance, are plotted in Fig 2. The theoretical curve derived from the THROE fitting with $(\gamma, q) = (1, 0)$, that gave the values in equation (6), is also shown, as a full line.

5 ANALYSIS OF ECCENTRICITY AT 29:2 RESONANCE

Before the changes in eccentricity due to resonance can be analysed, the values of eccentricity in Table 2 have first to be cleared of perturbations due to zonal harmonics, atmospheric drag and lunisolar perturbations.

The removal of zonal harmonic and lunisolar perturbations was performed by using the PROD⁵ computer program, and the variation due to drag in an atmosphere with day-to-night variation was calculated using the theory given in the Appendix of Ref 10 with scale height H taken as 106 km, appropriate to a mean height of 734 km. This theory assumes that the atmospheric density depends on the geocentric angular distance from the point of maximum density, which has been taken as 14 h local time¹¹.

In Fig 3a the values of eccentricity from Table 2 are given as circles, and these values after removal of zonal harmonic and lunisolar perturbations are indicated by triangles. The correction Δe_D for drag in an atmosphere with day-to-night variation is shown in Fig 3b. The values of eccentricity cleared of these perturbations are now ready to be fitted with equation (4) in integrated form, using THROE.

After the values of M_2 had been altered to mean values, \bar{M}_2 , as for the inclination, the first THROE fitting with $(\gamma, q) = (1, \pm 1)$ was performed and the value of ϵ obtained was 3.91. However, two of the eccentricity values, had standard deviations less than 0.000005; so both were increased to that value, in case there was a significant effect from the neglect of earth and ocean tides. Six values of eccentricity had to have their standard deviations increased by a factor of 2, and two by a factor of 4, to keep all the weighted residuals less than 2ϵ . The THROE run with these adjustments gave $\epsilon = 2.53$.

In some previous fittings of values of eccentricity with THROE, it was found that adjustment of the odd zonal harmonics was needed. The same method has been used here as in Ref 12, *ie* to add an increment ΔJ_3 to the J_3 value used in the PROD model. The lowest value of ϵ , 2.497, was obtained with $\Delta J_3 = -0.015 \times 10^{-6}$.

This final fitting of the values with $(\gamma, q) = (1, \pm 1)$ and with $\Delta J_3 = -0.015 \times 10^{-6}$, gave values of the lumped harmonics as follows:

$$\left. \begin{aligned} 10^9 \bar{C}_{29}^{-1,1} &= 53.4 \pm 3.8 & 10^9 \bar{S}_{29}^{-1,1} &= -10.3 \pm 5.6 \\ 10^9 \bar{C}_{29}^{-1,3} &= 18.9 \pm 6.9 & 10^9 \bar{S}_{29}^{-1,3} &= -3.9 \pm 7.8 \end{aligned} \right\} \quad (8)$$

The values of eccentricity, cleared of all known perturbations larger than the standard deviations of the values (except those due to resonance), are plotted in Fig 4. The fitted theoretical curve is that of the THROE run which produced the values in equation (8), and is shown as a full line.

6 INCLINATION AND ECCENTRICITY FITTED SIMULTANEOUSLY

The values of inclination and eccentricity fitted separately by THROE can be fitted simultaneously using the RAE computer program SIMRES developed by Dr R.H. Gooding. This program combines the results from a number of THROE runs (each with the same set of (γ, q) terms), and produces a single set of coefficients to fit the data. For this SIMRES fitting, the results of THROE runs with $(\gamma, q) = (1, 0)(1, 1)$ and $(1, -1)$ were used. The SIMRES program allows a choice of weighting, so that the contributing THROE runs can be given more or less weight according to their accuracy of fit, which is indicated by the value of ϵ .

The THROE fitting of inclination with $(\gamma, q) = (1, 0)(1, 1)$ and $(1, -1)$ gave $\epsilon = 0.927$, and for eccentricity the value of ϵ was 2.100, when fitted with the same terms. For the SIMRES fitting, therefore, the weighting of ϵ was downgraded by a factor equal to the ratio of the values of ϵ on the THROE fittings, namely 2.265 ($= 2.100/0.927$). The values of the lumped harmonics given by this SIMRES fitting are:

$$\left. \begin{aligned} 10^9 \bar{C}_{29}^{0,2} &= -2.7 \pm 0.5 & 10^9 \bar{S}_{29}^{0,2} &= 8.2 \pm 1.0 \\ 10^9 \bar{C}_{29}^{-1,1} &= 51.2 \pm 3.5 & 10^9 \bar{S}_{29}^{-1,1} &= -7.5 \pm 5.3 \\ 10^9 \bar{C}_{29}^{-1,3} &= 16.1 \pm 6.5 & 10^9 \bar{S}_{29}^{-1,3} &= -2.3 \pm 7.3 \end{aligned} \right\} \quad (9)$$

The SIMRES fittings of inclination and eccentricity are shown pictorially in Figs 2 and 4, as broken lines.

The lumped harmonics obtained from the fitting of inclination and eccentricity separately, given in equations (6) and (8), are very similar to those obtained from the simultaneous fitting, given in equation (9): all like coefficients are within one standard deviation of each other. So either set of values could be used in future determinations of the individual 29th-order harmonic coefficients in the geopotential. The SIMRES values are probably better, for two reasons. First, they take into account the $(\gamma, q) = (1, \pm 1)$ terms for inclination, and these may be significant because, when estimating the orders of magnitude of the terms (see section 3), it was found that neglect of the $(1, \pm 1)$ terms in fitting the inclination could give an error of 8%. Second, the SIMRES curve does seem to provide the better fit in Fig 2, especially between MJD 45650 and 45900, and ϵ is slightly lower (1.05 as against 1.08).

7 EQUATIONS FOR INDIVIDUAL COEFFICIENTS

The lumped harmonics in equation (9) can be expressed as linear sums of 29th-order individual coefficients: see equation (5). The Q coefficients in these equations depend⁴ on the ratios of the eccentricity functions $G_{\ell pq}$, and can be evaluated using the RAE computer program PROF. However, a correction factor has to be applied to the values of Q , as it was assumed in the PROF program that the e^2 terms in the functions $G_{\ell pq}$ could be neglected (see Ref 4). This leads to large errors in Q if e is large; but here, with e approximately 0.005, the correction is almost negligible, being less than 1%. The correction has, nevertheless, been made. The values of the lumped harmonics from THROE and SIMRES also need a very small correction, which has been made in equations (6) to (9).

The equations given below for the individual coefficients have been terminated when the expected contribution from the coefficients permanently falls to less than 5% of the largest contribution.

The resulting six equations are as follows:

$$\begin{aligned} \bar{C}_{29}^{0,2} = & \bar{C}_{30,29} + 0.044\bar{C}_{32,29} - 0.356\bar{C}_{34,29} - 0.349\bar{C}_{36,29} - 0.175\bar{C}_{38,29} - 0.000\bar{C}_{40,29} \\ & + 0.101\bar{C}_{42,29} + 0.122\bar{C}_{44,29} \quad , \end{aligned} \quad (10)$$

$$\bar{S}_{29}^{0,2} : \text{the equation is the same as (10) with } S \text{ instead of } C \quad , \quad (11)$$

$$\begin{aligned}\bar{C}_{29}^{1,1} = & \bar{C}_{29,29} - 1.576\bar{C}_{31,29} - 0.726\bar{C}_{33,29} + 0.244\bar{C}_{35,29} + 0.635\bar{C}_{37,29} + 0.524\bar{C}_{39,29} \\ & + 0.198\bar{C}_{41,29} - 0.099\bar{C}_{43,29} - 0.249\bar{C}_{45,29} - 0.246\bar{C}_{47,29} - 0.148\bar{C}_{49,29}, \\ & \dots\dots(12)\end{aligned}$$

$$\bar{S}_{29}^{1,1} : \text{ the equation is the same as (12) with } S \text{ instead of } C, \quad (13)$$

$$\begin{aligned}\bar{C}_{29}^{-1,3} = & \bar{C}_{29,29} - 0.590\bar{C}_{31,29} - 0.759\bar{C}_{33,29} - 0.464\bar{C}_{35,29} - 0.087\bar{C}_{37,29} + 0.183\bar{C}_{39,29} \\ & + 0.290\bar{C}_{41,29} + 0.258\bar{C}_{43,29} + 0.148\bar{C}_{45,29}, \quad (14)\end{aligned}$$

$$\bar{S}_{29}^{-1,3} : \text{ the equation is the same as (14) with } S \text{ instead of } C. \quad (15)$$

8 LUMPED HARMONICS FROM 1968-40B COMPARED WITH THOSE FROM COMPREHENSIVE GEOID MODELS

It is interesting to compare the values of lumped harmonics obtained here with those from comprehensive gravity-field models. Five models have been used for comparison: GEM 10B and GEM 10C¹³, the model produced in 1981 by R.H. Rapp¹⁴, GRIM 3-L1¹⁵ and GEM-T1¹⁶. The values of 29th-order lumped harmonics given by substituting the values of the individual coefficients from the models into equations (10) to (15) appear in Table 4.

The Goddard Earth Model GEM 10B extends to order and degree 36, and GEM 10C consists of the GEM 10B solution up to degree 36, together with some 31000 coefficients of order and degree up to 180, derived from analysis of altimeter measurements over the oceans. Equations (10) to (15) require coefficients above 36, some as high as degree 49, but the lumped harmonics in Table 4 for GEM 10B are truncated at degree 36. The expected accuracy of the GEM 10B coefficients has been assumed¹⁷ to be 3×10^{-9} . The GEM 10C accuracy (above degree 36) is certainly poorer¹⁸ and is rather arbitrarily taken as 5×10^{-9} . The standard deviations for the GEM 10B and GEM 10C values in Table 4 have been assessed using these accuracies for the individual coefficients.

The comprehensive geopotential model produced by R.H. Rapp at the Ohio State University in 1981 gives the individual coefficients to order and degree 180, and is derived from Seasat altimeter data, terrestrial gravity measurements and other data. An accuracy estimate for each coefficient is also given, so the standard deviations for the lumped harmonics in Table 4 have been assessed using

these accuracies. Finally, the lumped harmonics obtained by using the individual coefficients from GRIM3-L1 and GEM-T1 are given in Table 4. These models only extend to degree and order 36, so the equations (10) to (15) have to be truncated at this value, as with GEM10B. The standard deviations are calculated from the accuracies given for the individual coefficients in both cases.

Table 4

Values of lumped harmonics from 1968-40B and comprehensive geoid models

	$10^9 \bar{C}_{29}^{0,2}$	$10^9 \bar{S}_{29}^{0,2}$	$10^9 \bar{C}_{29}^{1,1}$	$10^9 \bar{S}_{29}^{1,1}$	$10^9 \bar{C}_{29}^{-1,3}$	$10^9 \bar{S}_{29}^{-1,3}$
1968-40B	2.7 ± 0.5	8.2 ± 1.0	51.2 ± 3.5	-7.5 ± 5.3	16.1 ± 6.5	-2.3 ± 7.3
GEM 10C	-0.5 ± 3	5.2 ± 3	25.9 ± 8	-12.5 ± 8	15.3 ± 5	-5.2 ± 5
GEM 10B	1.3 ± 3	4.9 ± 3	24.7 ± 6	-8.9 ± 6	14.4 ± 4	-5.4 ± 4
Rapp (1981)	-0.5 ± 1.9	7.1 ± 2.5	32.0 ± 4.2	-1.2 ± 6.5	13.3 ± 3.1	-11.7 ± 4.3
GRIM3-L1	0.8 ± 3.3	5.8 ± 3.3	34.5 ± 6.0	-10.3 ± 6.0	24.6 ± 4.3	-13.6 ± 4.3
GEM-T1	6.5 ± 6.2	2.7 ± 6.2	34.7 ± 10.4	11.6 ± 10.4	24.2 ± 8.8	8.0 ± 8.8

On examination of Table 4 it is seen that all the lumped harmonics from the models, except $\bar{C}_{29}^{1,1}$, agree well with those from 1968-40B, within 0.6 times the sum of their standard deviations on average; only the Rapp value for $\bar{C}_{29}^{0,2}$ and the GEM-T1 value for $\bar{S}_{29}^{1,1}$ differ from the 1968-40B values by more than the sum of their standard deviations. The values for $\bar{C}_{29}^{1,1}$ do not agree so well with that from 1968-40B: on average the difference is 2.1 times the sum of the standard deviations. The value of $\bar{C}_{29}^{1,1}$ from 1968-40B is large and well-defined, originating from the large and well-defined increase in eccentricity between MJD 45400 and 45700 in Fig 4.

The standard deviations of the values from 1968-40B are much better than those of the models for the first pair of coefficients, somewhat better for the second pair, and rather worse for the third pair.

The good agreement between the lumped harmonics in Table 4 is very satisfying, as it shows that the 29th-order coefficients in the models must be of the right magnitude and that their estimated errors are quite realistic. A previous analysis¹⁹ of the 29:2 resonance of 1967-104B with less accurate orbits led to similar though less definite conclusions. Hopefully in the future more satellites can be analysed at other inclinations as they pass through 29:2 resonance, and then the individual coefficients can be evaluated from satellite orbit analysis.

In conclusion, it is worth emphasizing that the results obtained here from 1968-40B are completely independent of the models: the gravity field used in PROP is from a much earlier model, of zonal harmonics and $J_{2,2}$ only; and 1968-40B is not among the satellites utilized in the models of Table 4.

9 APPROXIMATE ACCURACY IN GEOID HEIGHT

Equations (10) to (15) are useful in allowing an approximate assessment of the accuracy of the lumped harmonics evaluated for 1968-40B, in terms of an accuracy in geoid height. The accuracy σ_g in geoid height may be estimated approximately as $R\sigma/Q^*$, where σ is the error in the lumped harmonic and

$$Q^* = \left\{ \sum \left(\frac{Q_{\ell}^{q,k} \ell_0^2}{\ell^2} \right)^2 \right\}^{\frac{1}{2}}.$$

The values of σ_g for each lumped harmonic are as follows:

$\bar{C}_{29}^{0,2}$	$\bar{S}_{29}^{0,2}$	$\bar{C}_{29}^{1,1}$	$\bar{S}_{29}^{1,1}$	$\bar{C}_{29}^{-1,3}$	$\bar{S}_{29}^{-1,3}$
σ_g (cm)	0.3 0.6	1.2 1.8	3.1 3.5		

The values for $(q,k) = (1,1)$ are more accurate than those for $(q,k) = (-1,3)$, probably because the orbit determination covered the $(\gamma,q) = (1,1)$ resonance ($\dot{\phi} = \dot{\omega}$), but did not continue long enough to cover fully the $(\gamma,q) = (1,-1)$ resonance ($\dot{\phi} = -\dot{\omega}$). The accuracy of the first pair, $(q,k) = (0,2)$, is much better than has previously been obtained for order 29 from resonance analysis, or any other method (as Table 4 shows).

10 CONCLUSIONS

The orbit of 1968-40B has been determined at 74 epochs from some 6300 observations, between 1983 January and 1985 July while the satellite was passing through the condition of 29:2 resonance. The average accuracy of the inclination and eccentricity for all 74 epochs was equivalent to 110 m and 80 m in distance respectively.

The variations in inclination and eccentricity have been analysed, and six 29th-order lumped harmonics have been evaluated: the recommended values are those given in equation (9). The first two have standard deviations equivalent to accuracies of 0.3 and 0.6 cm in geoid height, considerably better than has

previously been obtained for order 29 from resonance analysis or any other method. The standard deviations of the other four values correspond to accuracies between 1.2 and 3.5 cm in geoid height. The results provide an independent test of the 29th-order harmonics from recent comprehensive gravity models: comparisons with five such models (Table 4) show good agreement and suggest that the estimated errors of the models are quite realistic, for order 29.

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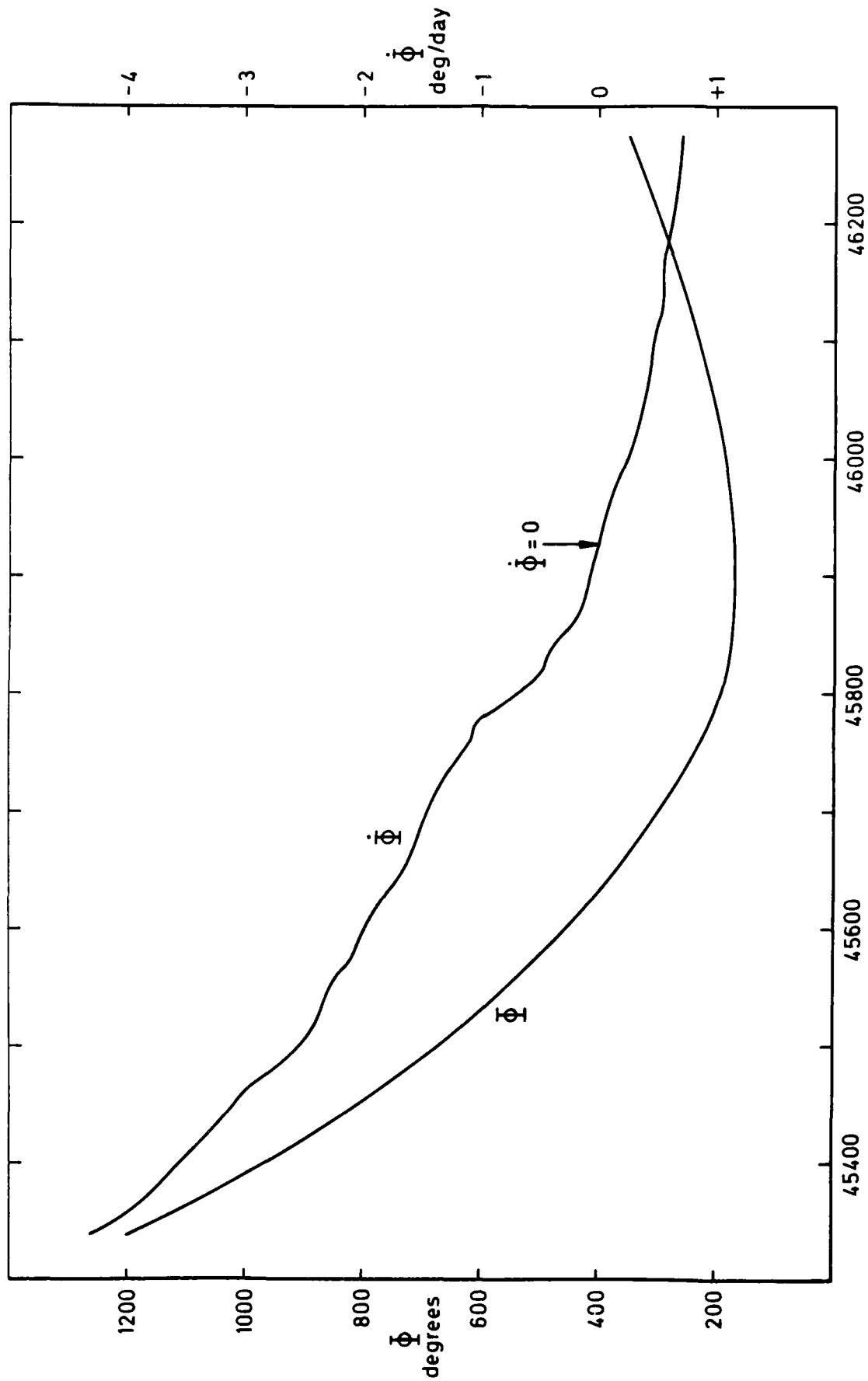


Fig 1 Variation of Φ and $\dot{\Phi}$ near 29:2 resonance

Fig 2

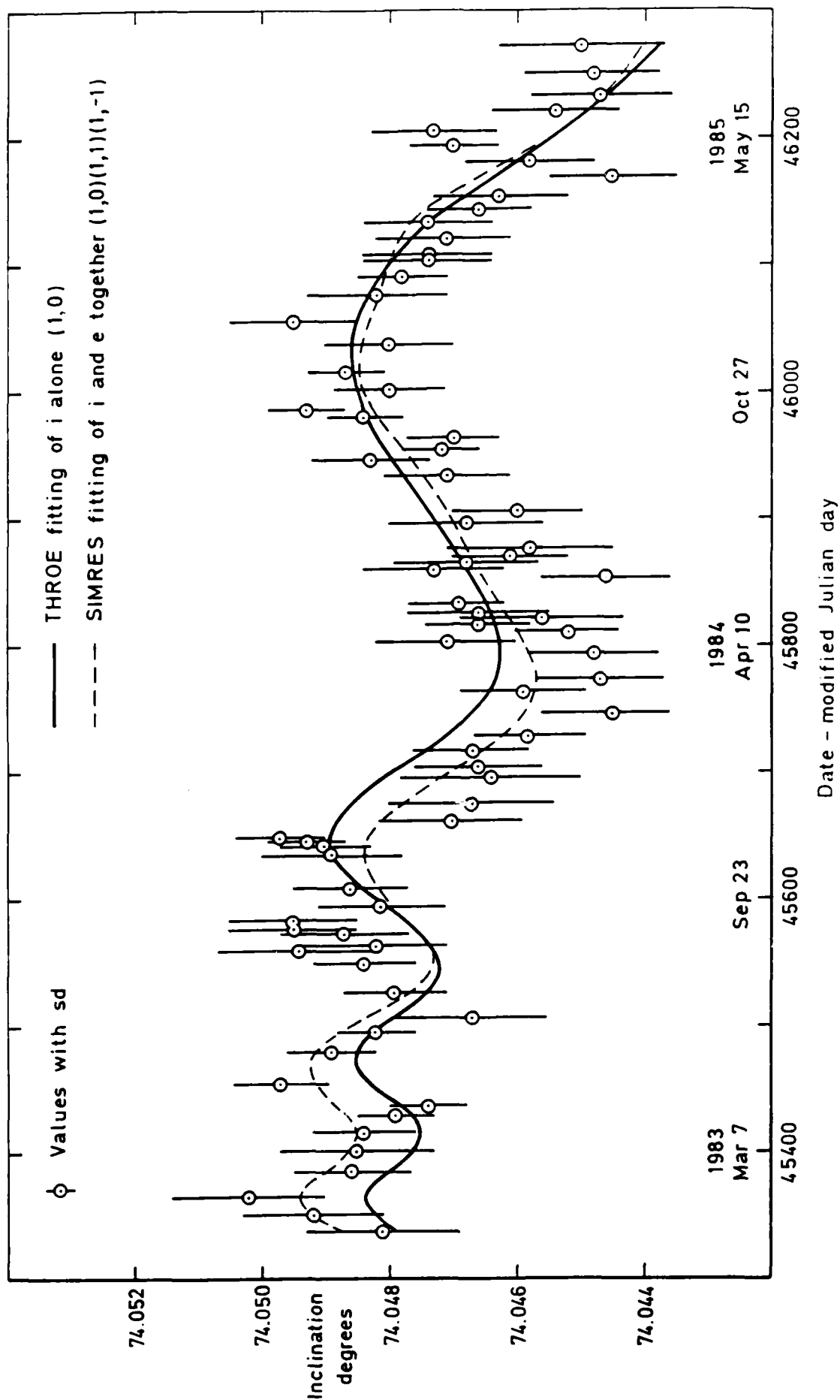


Fig 2 Values of inclination near 29:2 resonance, with fitted theoretical curves

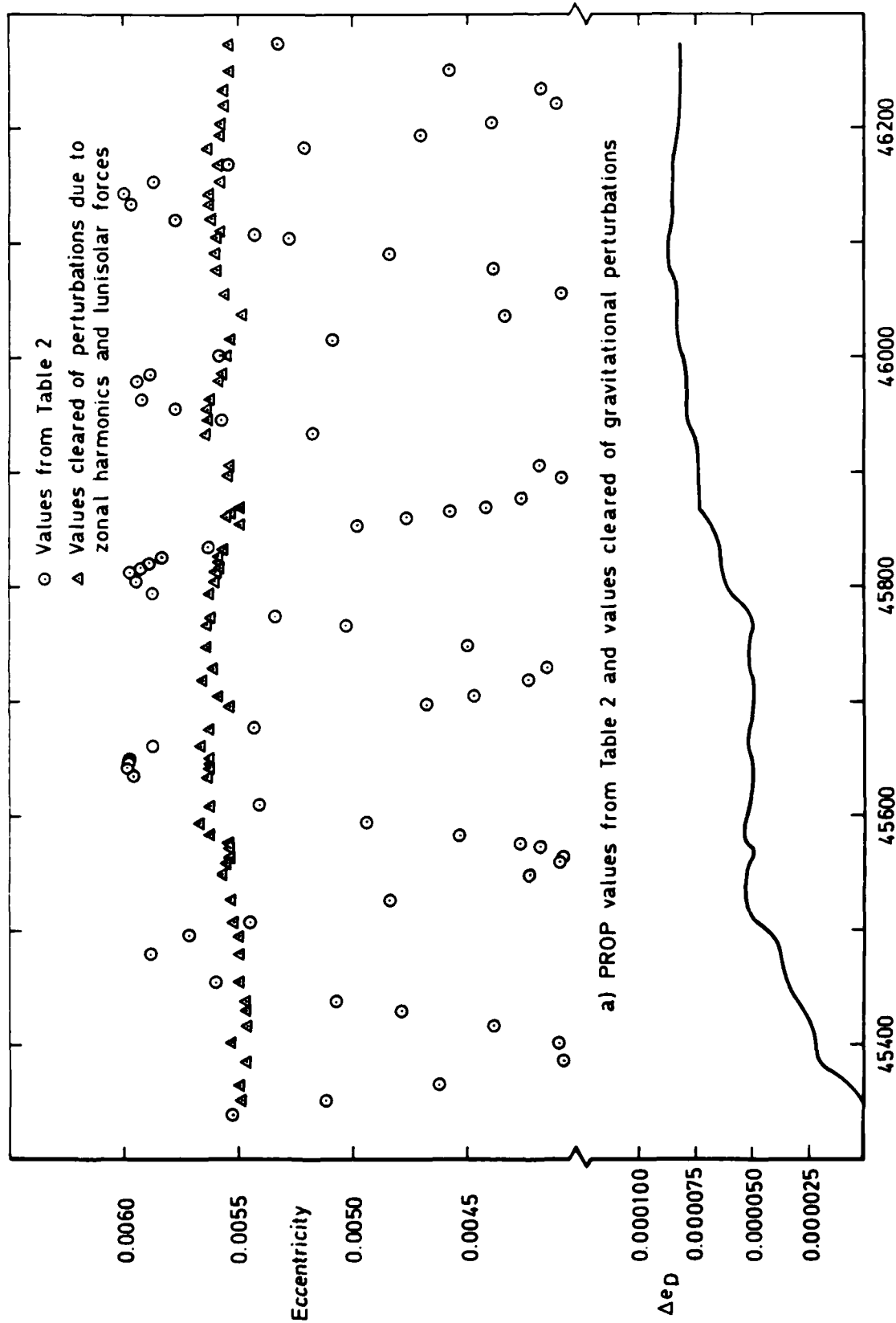


Fig 3

Fig 4

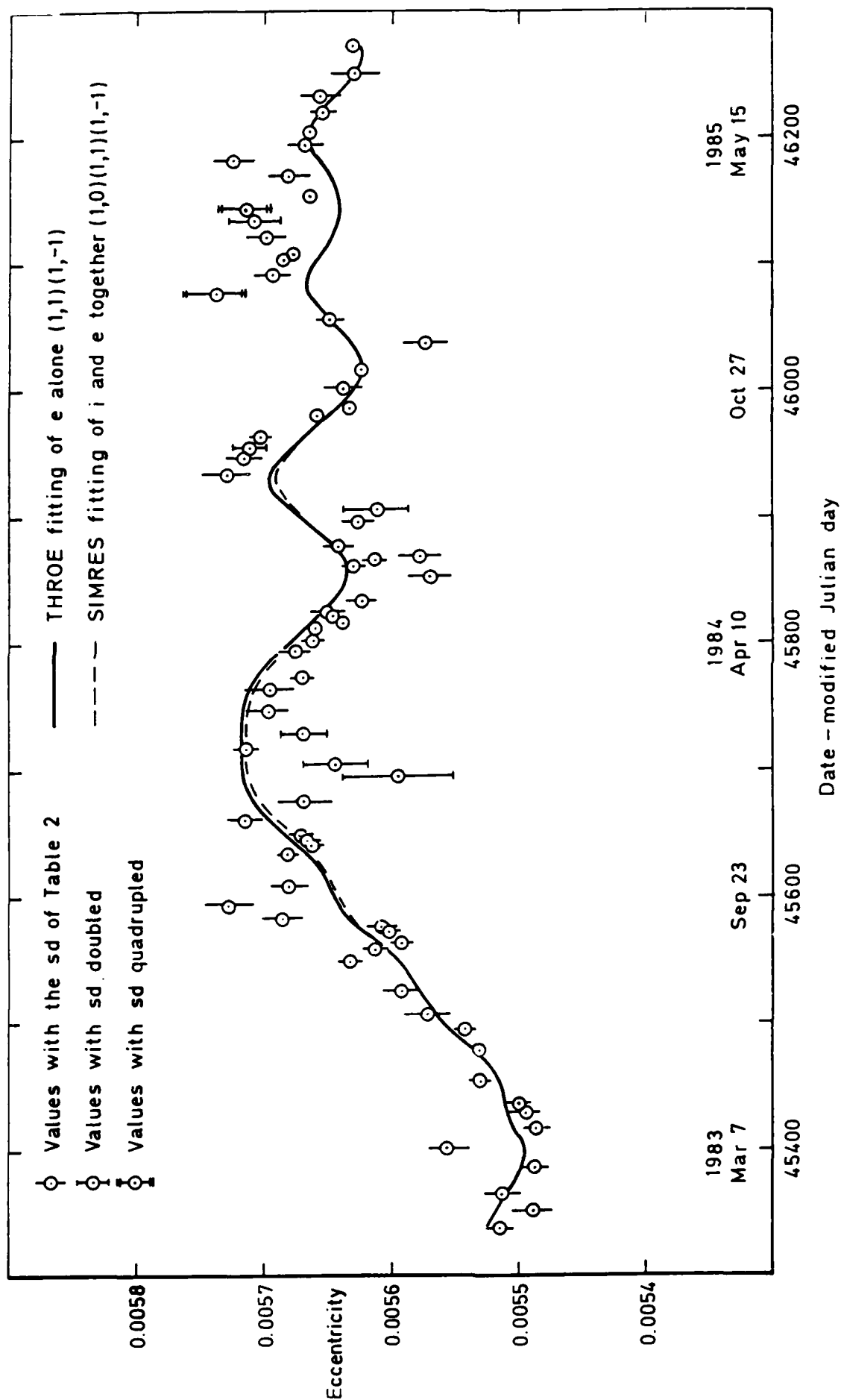


Fig 4 Values of eccentricity near 29:2 resonance, with fitted theoretical curves